the gas bubble, the spherical shock wave is reflected, and the bubble "explodes" from the inside, breaking up into small fragments. The collapse of the gas bubble or, more precisely, its shock destruction occurs. Gas pressure and temperature inside the bubble during the focusing and the subsequent reflection of the shock wave reach very large, albeit theoretically restricted, values [19]. When the collapse of the gas bubble is completed, its small fragments are left in the singled-out liquid volume, which are equal in size to the original cavitation nuclei, and the density of the singled-out liquid volume becomes close to the initial liquid density, ρ_f. As we show below, when the oscillation velocities of the ultrasonic radiators reach very high values, cavitation may follow a different mechanism, which does not involve breaking the gas bubbles up into small fragments, but rather exhibits bubble behavior approaching that of an empty Rayleigh cavity.

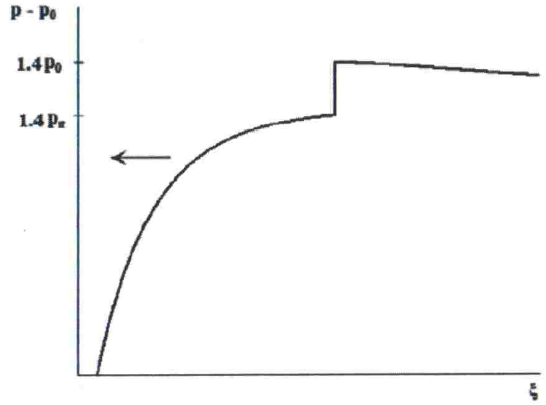

Figure 2. Instantaneous distribution of the excessive pressure in liquid near the cavitation bubble wall at $U > c_{min}$ is shown. The quantity $\delta p'_\infty$ is not taken into account.

This approach permits easily eliminating a seemingly clear contradiction that follows from the Noltingk-Neppiras equation: how can a gas-filled bubble implode with a very high rate if the gas pressure inside the bubble during compression rapidly increases, while the rate of the gas diffusion from the bubble, according to [4, 5], is negligible. In the proposed model, the gas bubble does not implode in the literal sense of the word, but is destroyed by a spherical shock wave reflected after focusing in its center. The presence of a well-known phenomena accompanying acoustic cavitation, such as sonoluminescence, erosion and dispersion of solids, emulsification of liquids, etcetera, can be well explained from this point of view. Additionally, the mechanism of the dissipation of the primary acoustic energy during the liquid cavitation becomes clear. This is the mechanism of the heating of a compressible medium in a shock wave, which is well described in the literature (see, for example, [19]).

2.3.2. Cavitation Region

During the rarefaction of a liquid in an acoustic wave, a mixture of a great number of spherical gas bubbles with the liquid (cavitation region) is formed. Let us call this gas-liquid mixture present in the cavitation region, the "continuum". In the previous section, the course of events during the collapse of a single bubble in some small volume of liquid was described. To extend these events over the entire continuum, a transition to spatial description is necessary. At that, the results of this transition must depend neither on the dimensions and the form of the continuum itself nor on the sizes and the spatial distribution of the bubbles in it.

During the compression stage, an acoustic radiator creates a pressure impulse in the liquid beyond the continuum in the form of a plane acoustic wave. Since the velocity of sound in the continuum is finite, the collapse of a multitude of gas bubbles located arbitrarily in the continuum must also occur simultaneously only in some narrow layer, as the impulse of the acoustic pressure approaches it, i.e. it must have a wave character. In the current model representation, the result of the superposition of many spherical shock waves, which are formed near each gas bubble during its collapse in a narrow layer of the continuum, is a spatial wave (SW) moving through the continuum. Such a representation is the most exact and visual way of extending the events occurring during a single gas bubble collapse, over the entire continuum.

In the real situation, the cavitation region in a liquid may take very complex, branched shapes. The spatial distribution of bubbles in the region also may be quite non-uniform and the sizes of the bubbles may vary. When the transition to the presented spatial description of cavitation is made, for the results to be independent of the shape of the cavitation region as well as of the spatial distribution and the sizes of the bubbles, in our initial equations we will further utilize hypothetical physical parameters related to the cavitation region as a whole. In other words, instead of operating with local values of density, changes in the internal energy and so on, we will use the values averaged over the whole cavitation region. As demonstrated below, these values disappear when further modifications of the fundamental equations are made.

The experimental investigations of acoustic cavitation described below conducted for the verification of the presented model were carried out using calorimetry of the entire environment and, therefore, provide only the spatially averaged values due to a relatively high thermal conductivity of the liquid. Therefore, the final purpose of the calculations following this model is the determination of a cumulative value of the changes in the internal energy of the environment, as a result of acoustic cavitation.

The spatial wave (SW) described above has a bore wave-like character, however, the continuum density and pressure inside the SW front change stepwise. This occurs because the cavitation bubbles collapse inside its front, following the process outlined in section 2.3.1. The presence of such a wave is the final stage of acoustic cavitation, within one cycle of the continuum rarefaction - compression. In other words, according to the model, it is assumed that the collapse of the gas bubbles occurs inside a relatively narrow front of a hypothetical SW, being formed and moving through the continuum in each compression half-period of an acoustic radiator.

The width of the SW front, inside which the collapse of the bubbles and the change of the continuum density occur, can be estimated as the product of the empty bubble collapse time, according to equation (3) and the wave front movement velocity with respect to the continuum, $h = c\tau$. A rough estimate for the wave front movement velocity can be made using expression (5). Then, at $\alpha = 0.1$ (taken from the literature data [22] and characteristic for the initial stage of acoustic cavitation) we obtain $h \approx 3r_{in}$. According to the estimation performed in the work [4], the maximum radius of a gas bubble in water does not exceed $2 \cdot 10^{-4}$ m, since larger bubbles rapidly rise to the surface. Hence, the value is: $h \leq 6 \cdot 10^{-4}$ m, which is smaller than the dimensions of the continuum itself by many orders of magnitude. Thus, the specified wave has a front that is very narrow relative to the dimensions of the entire continuum. Getting over this barrier, therefore, the physical parameters of the continuum change stepwise.

It is necessary, further, to establish a relation between the continuum parameters ahead of and behind the SW front, as well as the relationship between these parameters and the oscillatory velocity of an acoustic radiator. It is important to note that the velocity of the specified wave can be lower than the velocity of sound in the continuum.

The SW moving through the continuum is not only a physical abstraction used for the construction of the model, but can, apparently, exists in reality. In this case, however, we are not faced with an ordinary shock wave, which arises in a compressible continuum when the piston movement velocity is higher than the sound velocity in the continuum. Such shock waves in a gas-liquid suspension obtained by bubbling a gas through a liquid are described in detail in literature [21]. Here, it is assumed that in a gas-liquid suspension formed as a result of the liquid rarefaction in an acoustic wave, another type of bore wave-like shock waves may exist, which is associated with the radial movement of the liquid in the vicinity of each bubble.

It is well known that when a jump (discontinuity) of a physical quantity arises in a compressible continuum, a solution should be sought using the general conservation laws in the form of the Rankine-Hugoniot equations [19]. These equations reflect the ratios of the steady-state physical parameters of the compressible continuum before and after the passage of the shock wave front. Additionally, there appears a possibility to analytically calculate the values of important parameters, without considering in detail the transient processes inside the SW front, which are connected with the complex kinetics of a collapsing gas bubble.

Let us introduce the following designations: p_h is the pressure in the liquid phase of the continuum near the bubble wall after the SW passage; p_l, $\rho_l = \rho_f (1- \alpha_l)$, α_l are, respectively, the pressure in the liquid phase of the continuum near the bubble wall, the density and the volumetric gas content of the continuum before the SW passage. A scheme of the continuum flow is presented in Figure 3. It is assumed that a SW moves through the continuum, and that the gas bubbles collapse inside the narrow front of this wave. Also shown in this figure is the supposed pressure profile in the continuum.

Figure 4 shows the supposed processes occurring in one cycle of the acoustic cavitation of liquid. The pressure in the liquid phase of the continuum near the gas bubble wall in an arbitrary state is plotted on the ordinate, and the continuum specific volume is plotted on the abscissa. Line 1 represents the rarefaction of the continuum with cavitation nuclei in an acoustic wave. Line 2 represents a nonlinear process of the growth of cavitation bubbles in the rarefaction wave. Line 3 represents a preliminary compression of the continuum in an acoustic wave (for a single gas bubble, this corresponds to a rise in the gas pressure in the bubble on the smooth section of a converging spherical wave, as described in section 2.3.1). Line 4 represents the continuum's transition from one state to the other when the SW passes (for a single gas bubble, this corresponds to a rise in the gas pressure in the bubble on the steep section of a converging spherical wave, as described in section 2.3.1). In this scheme, it is assumed in advance that the velocity of the SW movement through the continuum can be lower than the sound velocity in the continuum itself ahead of SW. Additionally, the SW front itself serves as a source of the acoustic wave, propagating forward in the direction of the shock wave movement. In this connection, there is a preliminary compression of the continuum, and line 4 begins above the abscissa axis.

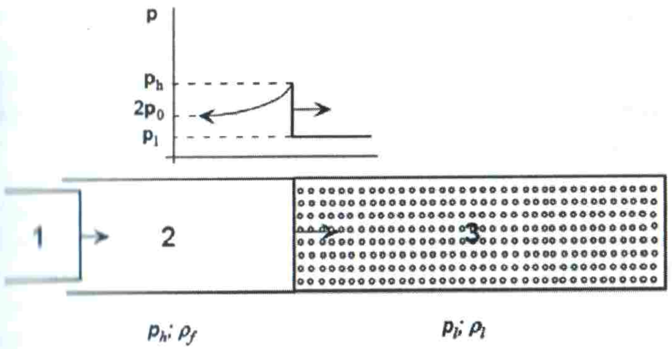

Figure 3. Schematic of the continuum's flow during compression is shown (1 – acoustic radiator, 2 – flow region after the SW passage, 3 – flow region before the SW passage).

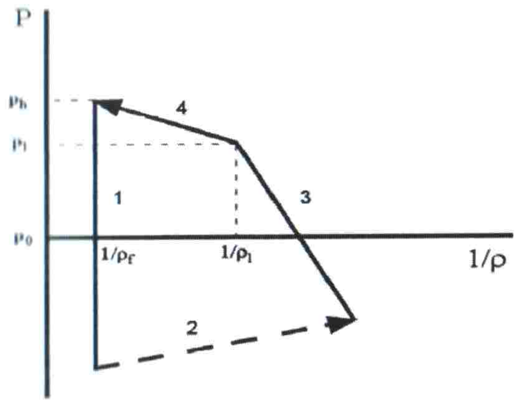

Figure 4. Processes occurring during acoustic cavitation are illustrated. Line 1 represents the rarefaction of the continuum with cavitation nuclei in an acoustic wave, line 2 represents a nonlinear process of the growth of cavitation bubbles in the rarefaction wave, line 3 represents a preliminary compression of the continuum in an acoustic precursor wave, line 4 represents the continuum transition from one state to the other when the SW passes.

This kind of an acoustic wave is called a precursor. The precursor does not cause the collapse and disintegration of the bubbles because of a small value of its amplitude. Similar representations are used for initially loose or porous environment. In such environment, during the compression phase, the shock-wave front is formed only due to the parameters of the compression process itself since this environment tends to change the specific volume of pores (cavities) abruptly (stepwise) under pressure [23-25].

Let us introduce the following additional designations: $p_l = p_0 + p'_l$, $p_h = p_0 + p'_h$; p'_l and p'_h are the excessive pressures in the liquid phase of the continuum near the bubble wall before and after the SW passage, respectively; u_l and u_h are the continuum flow velocities relative to SW before and after its passage, respectively; e_l and e_h are the specific internal energy of the continuum before and after the SW passage, respectively; v is the current oscillatory velocity of an acoustic radiator; v_t is the critical oscillatory velocity of an acoustic radiator, which corresponds to the cavitation onset (cavitation threshold). Note that a stepwise increase in the continuum density from ρ_l to ρ_f at the SW front corresponds to a change in pressure from p_l to p_h. The relative movement of the liquid and the gas bubbles is neglected.

Let us now write the system of conservation equations (Rankine-Hugoniot equations) for the continuum parameters on both sides of the SW front:

$$\begin{aligned} &\rho_l u_l = \rho_f u_h, \\ &p'_l + \rho_l u_l^2 = p'_h + \rho_f u_h^2, \\ &\frac{p_0 + p'_l}{\rho_l} + \frac{u_l^2}{2} + e_l = \frac{p_0 + p'_h}{\rho_f} + \frac{u_h^2}{2} + e_h, \\ &v - v_t = u_l - u_h \end{aligned} \quad (7)$$

The fourth equation of system (7) shows that a change in the continuum's movement velocity getting over the SW front is equal to the excessive oscillatory velocity of an acoustic radiator, which exceeds the critical value, v_t.

This system of equations can be transformed to the following form:

$$I = \frac{(2p_0 + p'_l + p'_h)}{2}(v - v_t), \quad (8)$$

$$\eta_l = \frac{(v - v_l)^2}{p'_h - p'_l}$$

Here, $I = (e_h - e_l)\rho_l u_h$ is the flux density of the energy dissipated inside the SW as a consequence of the dissipation processes related to the bubble collapse and $\eta_l = \alpha_l/\rho_l$ is the volume of all cavitation bubbles per unit mass of the liquid phase of the continuum before the SW passage.

The average flux density of the acoustic energy (acoustic energy intensity) absorbed in one acoustic wave period can be presented in the following way:

$$I_a = \frac{\omega}{2\pi}\int_0^{\pi/\omega} \left| I\, sin(\omega t)\right| dt = I/\pi \qquad (9)$$

2.4. Set-up of Equations for Experimental Verification

For the resulting equations (8) to be verified experimentally, it is necessary to determine the particular values of p'_h, p'_l, η_l and v_l.

2.4.1. Low Oscillatory Velocities of Acoustic Radiator

From equation (6) and the analysis given in section 2.3.1, it follows that the maximum excessive pressure at the SW front is equal to $p'_h = 1.4p_0 + \delta p'_\infty$. As mentioned above, the liquid utilized for the construction of the theoretical model, does not possesses tensile strength during rarefaction. Consequentially, the explosive growth of the cavitation nuclei and their conversion into gas bubbles in the rarefaction wave takes place at the negative pressure equal to the static pressure, $p'_\infty = p_0$. It is possible to assume that for small oscillation velocities of the acoustic radiator near the cavitation threshold a symmetry of acoustic pressure amplitudes during the half periods of compression and rarefaction is conserved. Consequentially, in this case, $\delta p'_\infty = 0$ and $p'_h = 1.4p_0$. It will be shown below that for large radiator oscillatory velocities it is no longer possible to ignore the quantity $\delta p'_\infty$. Note that the value of $p'_h \approx 1.4p_0$ actually corresponds to the threshold of water cavitation, at least, in its initial stage. This fact was experimentally established in [26].

Above, it was assumed that during the rarefaction of a liquid in an acoustic wave, all gas dissolved in a unit volume of the liquid passes into the bubbles formed in this volume. The oscillations of the gas bubbles before the onset of their collapse are isothermal, and the mass of the gas in them does not change. From the analysis of equation (6) given in section 2.3.1, it follows that $p'_l = 1.4p_g$, hence, the condition $p_0\eta_0 = 0.71p'_l\eta_l$ must be met. Here, η_0 is the equilibrium volume of gas dissolved in a unit mass of the liquid at the pressure, p_0.

The quantity v_t is the critical oscillatory velocity of an acoustic radiator, which corresponds to the cavitation threshold. In view of the conditions described above, one can assume that for a plane acoustic wave, $(v_t)_{rms} = 0.71p'_\infty/\rho_f c_f = 0.71p_0/\rho_f c_f$.

It should be borne in mind that the value of v_t in each particular experimental case can be different from the specified theoretical value. This is connected with the fact that the practical value of v_t depends on a large number of different parameters of liquid (physical nature, purity degree, gas content, volatility, sample preparation history, etc.). Besides, v_t also depends on the conditions of the conducted measurements (frequency of ultrasound, degree of isolation from external radiation, temperature, etc.)

From the second equation of system (8) we obtain:

$$p'_l = \frac{1.4p_0^2\eta_0}{\eta_0 p_0 + 1.42(v - v_t)^2_{rms}} \qquad (10)$$

Now from the first equation of system (8) in view of equations (9, 10) we obtain the final equation for the average flux density of the acoustic energy (intensity of acoustic energy) absorbed in the cavitation region:

$$I_a = 0.76p_0\left[1 + \frac{0.41p_0\eta_0}{\eta_0 p_0 + 1.42(v - v_t)^2_{rms}}\right](v - v_t)_{rms} \qquad (11)$$

For the initial stage of acoustic cavitation, at a small value of $(v-v_t)_{rms}$, the final equation is as follows:

$$\frac{I_a}{p_0} = 1.07(v - v_t)_{rms} \qquad (12)$$

It is important to point out that in equations (11, 12) the quantities related to the spatial distribution of gas bubbles in the continuum and their size, as well as the form and shape of the continuum itself are not present.

2.4.2. High Oscillatory Velocities of Acoustic Radiator

From the main system of equations (7), one can obtain the expression for the SW velocity relative to the unperturbed continuum, $u_l = \left[(p'_h - p'_l)/\rho_f \alpha(1-\alpha)\right]^{0.5}$. The ratio of u_l to the sound velocity, c, in the continuum according to equation (5), using equation (10) and taking into account that $p_g = 0.71p'_l$, can be written as:

$$\frac{u_l}{c} = \left(\frac{p'_h - p'_l}{p_g}\right)^{0.5} = \left(\frac{2(v - v_t)^2_{rms}}{p_0 \eta_0}\right)^{0.5} \qquad (13)$$

From this expression, it is seen that at $(v\text{-}v_t)_{rms} \geq 1$ m/s, the SW movement must become supersonic, making it a real shock wave in the classical sense. When the SW movement is supersonic, a precursor is absent because it is absorbed by the faster shock wave. The density and the pressure of the gas inside the bubbles in this case are initially small since they are not compressed beforehand by the precursor. From the analysis of equation (10), it is seen that at $(v\text{-}v_t)_{rms} > 3$ m/s the gas pressure in such bubbles becomes approximately an order of magnitude lower than the static pressure, p_0, and continues to decrease. A spherical shock wave in rarefied gas inside such a bubble is not formed and, accordingly, the bubble does not break up into small fragments as a result of the collapse. The behavior of the bubble becomes close to the behavior of an empty Rayleigh cavity.

It is also important to keep in mind that the minimum width of the shock wave front in a gas is on the order of the molecule free path [19]. At a normal density of the gas, this distance is about 10^{-7} m. With a decreasing gas density, this distance increases proportionally and becomes close to the characteristic size of the bubble itself 10^{-5} m. Under these conditions, a spherical shock wave inside the bubble cannot be formed, and the bubble is compressed like a Rayleigh cavity.

At the final stage of the collapse of the bubble, the gas pressure in it increases to such a degree that it can hold back the liquid's pressure. At that, the pressure and temperature of the compressed gas can reach very high values (theoretically unrestricted under the assumptions of this model [19]). In this case, at the excess pressure, $p'_h = 1.4p_0$, the continuum behind the SW is a gas-liquid suspension with some density $\rho_h = \rho_f(1-\alpha_h)$. If the conditions identified in the beginning of section 2.3, assumed for the construction of the model, are to be met, the continuum behind the front of SW is additionally compressed by the acoustic radiator until density ρ_f is reached. This corresponds to a pressure increase at the SW front up to the value of $p'_h = 1.4p_0 + \delta p'_\infty = 1.4p_0 + 0.5c_h^2\delta\rho = 1.4p_0 + 0.5c_h^2\rho_f\alpha_h$, where $\delta\rho = \rho_f - \rho_h = \rho_f\alpha_h$ is the additional increase in the continuum's density behind the SW front, necessary to reach the quantity ρ_f, and c_h is the speed of sound in the gas-liquid suspension with density ρ_h. For high oscillatory velocities of acoustic radiator similar to the sound speed in the continuum, $p'_h = 1.4p_0 + \rho_f\alpha_h v^2_{rms}$, since in this case it can be taken that $c^2 = 2v^2_{rms}$.

The value of v_t is neglected. Since $\delta p'_\infty$ should be taken into account only at high v and the second term of equation (11), which corresponds to the excessive pressure p'_l, is negligible, we leave it unchanged. Let us now write equation (11) in the final form in view of equation (9):

$$I_a = 0.76p_0\left[1 + \frac{0.41\eta_0 p_0}{\eta_0 p_0 + 1.42(v-v_t)^2_{rms}} + \frac{0.29\rho_f\alpha_h v^2_{rms}}{p_0}\right](v-v_t)_{rms} \qquad (14)$$

2.4.3. Interpretation of Experimental Results of Work [26]

A large series of experiments aimed at studying acoustic cavitation of water at low oscillatory velocities of acoustic radiator is presented in the work [26]. Experiments were conducted in degassed water with the concentration of the dissolved air equal to 30% of the nominal concentration in the equilibrium state at the room temperature and the normal static pressure.

For the interpretation of these data, let us introduce the following designations: $\Sigma I_a = 0.5(p'_h)^2\gamma = p_0^2\gamma$ is the total intensity of the acoustic energy radiated into water; $I_{a0} = 0.5(p'_h)^2\gamma_f = p_0^2\gamma_f$ is the intensity of the acoustic energy propagating beyond the bounds of the cavitation region.